Innealta CAPITAL

COMMENTARY: SEPTEMBER 28, 2012

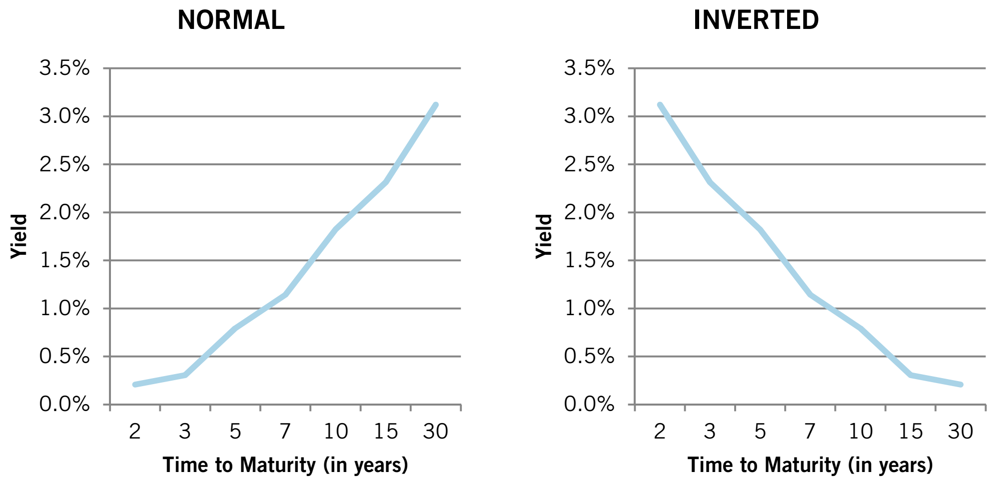
TERM STRUCTURE MODELS

- Giving a pause to allow the election cycle to fester a bit longer before we again revisit its ramifications, this month we return our attention to the fixed income market and provide further background to our readers on the theory and mechanics behind our approach to our fixed income portfolio allocations.
- Our objective is to provide the reader with a general overview of the mathematics and models used in the practice of managing fixed income portfolios. We build on our August commentary, which introduced the reader to interest rate risk management. That discourse touched upon the need for interest rate models and the usefulness of binomial lattices for interest rate risk evaluation and management. We extend the discussion here by focusing on the important role of interest rate models in pricing and risk management. As the vast majority of fixed income instruments today contain embedded option features, such that the timing and/or magnitude of the claims' cash flows are influenced by forward interest rates, interest rate models are of critical importance to fixed income portfolio managers.
- This month, we present a summary of no-arbitrage term structure models, which are critical tools capturing the relationship between interest rates, expected inflation and monetary policy actions. We focus on no-arbitrage interest rate models, structuring the discussion to progress from introducing the most simplistic to the most sophisticated and credible models. We emphasize the relative advantages of each incremental model, keeping in mind that all models are simplifications of reality and thus suffer from some imperfections.
- In highlighting this class of interest rate models, we seek to demonstrate to our readers and clients some of the tools we use to understand how the macroeconomic actions, from monetary policy to fiscal uncertainty, propagate their way through the system and ultimately manifest in the term structure—and, ultimately, the bond valuations—through the interpretations of these actions by other market participants. An intended byproduct of this series of missives, too, is the understanding by readers that Innealta is well versed in navigating these waters.

TERM STRUCTURE OF INTEREST RATES

Among our recent commentaries, the August 2012 volume highlights the role that interest rates, and in particular, the term structure of interest rates, play in fixed income portfolio management. Specifically, the term structure of interest rates depicts interest rates corresponding to various maturities. Embedded in the term structure are forward rates, which are periodic interest rates for time periods beginning in the future. Forward rates may be thought of as the force that pulls the term structure up or down. Long-term rates tend to be higher than short-term rates. A positively sloped term structure is thus considered "normal." The forward rates, periodic rates for future time periods, must be higher than the current one-period interest rate, and thus pull the term structure up. Conversely, an "inverted," or negatively sloped term structure, indicates decreasing forward rates. Exhibit 1 highlights this rather obvious distinction.

Exhibit 1: "Normal" and "Inverted" Term Structures



SOURCE: Innealta Capital

Practitioners use the term structure of interest rates for critical tasks, including bond pricing and risk management. On the surface, pricing a bond using the term structure of interest rates involves simply discounting the bond's obligated cash flows by the rate on the appropriate term structure corresponding to the time of the cash flow. In practice, however, this procedure is somewhat limited in scope as it applies only to "straight bonds," or those which do not contain embedded option features that alter the magnitude and/or timing or the bond's cash flows. Examples of bonds with embedded options are callable bonds (bonds whose issuer has the right to force redemption prior to maturity at an agreed upon price), putable bonds (bonds whose holders have the right to force the issuer to repurchase the bonds at a future date at an agreed upon price), and asset-backed-bonds where the collateral (such as mortgage loans) produces future cash flows of uncertain magnitude and timing where both are influenced by future interest rates. Critical to the valuation of bonds containing features that result in uncertainty over the timing and magnitude of future cash flows is a model depicting possible forward rates.

Interest rate models provide critical tools to fixed income professionals. These models enable the valuation of interest rate contingent claims, such as bonds with embedded option features, and evaluation of interest rate risk. Accurately depicting the term structure's movements through time, accounting for all possible interest rate scenarios, is a herculean task, a fact that no doubt gives rise to the myriad of interest rate models. No model can capture the complete dynamics of all of the many interrelationships across the fixed income markets, which is to say that even the most sophisticated models are abstractions from reality. In practice, parsimony and theory often dictate selection of the optimal model.

Interest rate models range from the fairly simplistic to the very complex. The defining features of each model come from their assumptions about the process generating innovations to (changes to) interest rates through time. Our focus in this commentary will be on one class of interest rate models referred to as "no arbitrage" interest rate models, which we believe provide the best representation of capital markets at any point in time. These models have the desirable property that they are fit to the current term structure so that the possible future interest rate paths generated by the model produce rates that, when used to price bonds, result in bond prices consistent with current market prices. Other classes of models, such as equilibrium models, begin with an interest rate process and proceed to derive conditions under a general equilibrium framework. An undesirable feature of equilibrium models is that they may result in prices inconsistent with current market prices, which contributes to our reliance on no-arbitrage models of the term structure. At Innealta, we utilize our extensive knowledge of term structure models in our daily management of our fixed income allocations. We present the no-arbitrage models here for educational purposes and to ensure that our readers and investors understand our adeptness at modeling the term structure and quantifying expectations of inflation in our portfolios.

INTRODUCTION TO NO-ARBITRAGE TERM STRUCTURE MODELS

Assumptions regarding interest rate dynamics—that is, how rates move or propagate forward—are the defining characteristics of each term structure model. The models describe the dynamics of the interest rate evolution through time through stochastic differential equations (SDEs). The specific SDE chosen represents a tradeoff between tractability of numerical solutions and the generality required to capture salient properties of interest rates, such as mean-reversion. While more complex SDEs allow the models to incorporate more features of interest rate dynamics, analytical tractability, as we will see shortly, suffers exponentially with increasingly complex SDEs.

Another key difference between interest rate models is in the chosen number of "factors" specified. To illustrate, in a one-factor model, the focus of our attention here, the short rate of interest (one period interest rate) determines the entire term structure. Other, more complex models include additional factors, specifying separate SDEs for each factor: both the short rate and a longer term rate, for example. We must emphasize that the solutions become numerically onerous with additional factors as there are simply insufficient pure arbitrage relationships in the capital markets to reach a solution across additional dimensions. Thus, although multi-factor models incorporate additional dimensions and result in elegant solutions in theory, numerical solutions are not possible without imposing additional assumptions. This fact contributes directly to multifactor models' practical limits.

We proceed to present several of the most popular no-arbitrage interest rate models, focusing on the important characteristics of each model and emphasizing that these assumptions drive the properties of the model solutions, making it imperative that the practitioner employing these models be intimately familiar with their assumptions.

The Ho-Lee Model: Pioneering No-Arbitrage Interest Rate Model

We begin with the model of Ho and Lee first as it was the first no-arbitrage interest rate model.[1] The Ho-Lee model assumes the short rate follows the following stochastic process:

[1] Thomas Ho and Sang Lee, "Term Structure Movements and Pricing Interest Rate Contingent Claims," Journal of Finance (1986) pages 1011-1029.

$$dr = \theta(t)dt - \sigma dz$$

Though the above equation might appear mathematically challenging, it simply states that the (instantaneous) changes in the one-period interest rate come from two sources: a drift and a random, or stochastic, component. The term $\theta(t)$ is the drift, or expected change, portion of the process. The magnitude of the drift is proportional to time, as dt refers to the passage of time. The drift component of the SDE determines important characteristics of the model. The values of the drift term come from the current term structure; they are results from the process of fitting the model to the term structure to ensure bond prices obtained through the interest rate model match observed bond prices (i.e. imposing no-arbitrage conditions). Said in another way, the values of the parameters (the size of the drift in this case) are determined such that prices of option-free bonds computed from the term structure model match observed market prices. This is a critical step since it ensures that the model rates accurately reflect bond market conditions. This insight is also critical to the computation of numerical solutions: absent the imposition of the no-arbitrage conditions, the models would suffer from the undesirable property of having multiple possible solutions, all but one of which would have no connection to market conditions. Thus, we cannot overstate the critical importance of the no-arbitrage property of term structure models.

The second component contributes the risk, or uncertainty. The stochastic component drives the distributional characteristics of the interest rate. The Ho-Lee model assumes the short-rate's level of uncertainty remains constant through time, which is why σ is not a function of time. Sparing the reader a discourse on stochastic calculus, we point out that the term dz is random and continuous. In short, the Ho-Lee model depicts the short rate as distributed normally. In fact, the normality assumption results in possible negative interest rates in the Ho-Lee model. Essentially, since the normal distribution is symmetrical, in the Ho-Lee model, possible future rates are distributed normally (symmetrically around the drift). The implication of this property is that, given sufficient volatility, or low interest rates, it is common for the Ho-Lee model to produce negative rates. Of course we recognize this weakness, but present the Ho-Lee model here as a way to introduce the reader to the first and most simplified no-arbitrage model before turning to incrementally more complex models. Keep in mind that this is the most simplistic model, making the fewest assumptions about the short rate and producing the most tractable solutions. The incremental models add additional features to address some of the undesirable outcomes, but with those additional features come increased difficulty in obtaining numerical solutions.

The process of fitting an interest rate model to the current term structure involves the use of complex numerical algorithms that reach well beyond the scope of our commentary. The investment team at Innealta has published extensively in the area of term structure modeling and fixed income portfolio management. Of particular relevance in this context, we refer the reader to a "how-to" manual for practitioners fitting these models.[2] We will, however, restrict to simply highlighting the critical step in the solution, which is to obtain an approximation for the interest rate, and in turn use that approximation to describe the movement of the interest rate from time k to time $k+1$. For the Ho-Lee model, this approximation is:

$$r_{k+1} = r_k + \theta_k \tau + \sigma_k \Delta z_k$$

To translate the math to English, the above equation simply describes the change in the interest rate from time k to time $k+1$ as equal to a drift term plus an uncertain risk component. We see that the drift term, $\theta_k \tau$, may grow large over time, since the drift component is the product of the drift rate θ_k can be and time elapsed τ. Additionally, for large volatilities, σ, innovations in the short rate may be influenced heavily by the stochastic component. These properties lead to two undesirable properties of the Ho-Lee model: The interest rate may exhibit unbounded growth, and negative interest rates are possible and quite common. In normal times,

[2] Buetow, Gerald and James Sochacki, "Term Structure Models Using Binomial Trees," Research Foundation of AIMR (2001).

negative interest rates are unlikely and an undesirable property of the solutions. In today's (insane) monetary policy environment, however, negative rates seem entirely possible.

For interest rate models to be useful to practitioners, it is valuable to have a tractable representation of the possible interest rates under the models. Thus, the models are often fit to binomial lattices. We thus turn our attention to presenting the Ho-Lee model represented on a binomial tree, depicting possible forward rates. The numerical techniques required to fit the models to binomial lattices include conditions ensuring bond prices obtained from the possible forward rates match the current term structure. The tree describes possible evolutions of the short rate according to the model of Ho and Lee. In a binomial lattice, the interest rate may make one of two possible moves over discrete time periods. We present the models where each time step corresponds to a semi-annual time period. Sparing the reader the details involved with "fitting" the interest rate models to binomial lattices, we proceed by presenting the end results. Exhibit 2 presents the Ho-Lee binomial lattice assuming the term structure is flat at 2.25% and volatility is constant at 1%.

Exhibit 2: Ho-Lee Binomial Lattice: Flat Term Structure (2.25%), 1% Vol.

				4.39%
			3.67%	
		2.96%		2.97%
	2.25%		2.26%	
		1.55%		1.56%
			0.84%	
				0.14%
Time in Years	**0**	**0.5**	**1**	**1.5**

SOURCE: Innealta Capital

Referring to the exhibit, the bottom line indicates time, measured in years. Time 0 corresponds to the six-month period beginning today. The next time, 0.5, refers to the six-month period commencing one year from today. As can be seen from the exhibit, according to the Ho-Lee model, the current one-period interest rate, which is known today, is 2.25%. The rate for the next period may evolve to either a higher or lower rate. The magnitude of the rate moves depends on the volatility. Under 1% volatility, the rate may move either up to 2.96% or down to 1.55%. For the third time-step, there are two possible moves (up and down) from each of the two possible rates at the preceding node. Note that the tree "recombines," meaning that should the interest rate move down the next period and up in the subsequent period, the same rate presents as if the rate had taken the path of up and then down. The Ho-Lee binomial lattice also illustrates the important property that the model assumes the interest rate is distributed normally. This is evident from the dispersion of the possible rates at each time step: the top and bottom nodes are spread roughly equally above and below the most central node on the lattice. Another important property of the Ho-Lee model is the tendency of the short rate to grow unboundedly. The lattice we present in Exhibit 2 contains too few time steps for this to present clearly, but extending the model forward illustrates this property.

Exhibit 3: Ho-Lee Binomial Lattice: Flat Term Structure (2.25%), 2% Vol.

				6.55%
			5.11%	
		3.67%		3.72%
	2.25%		2.28%	
		0.85%		0.90%
			-0.55%	
				-1.93%
Time in Years	**0**	**0.5**	**1**	**1.5**

SOURCE: Innealta Capital

To illustrate the role of volatility, we present the Ho-Lee binomial lattice again in Exhibit 3, where the term structure is flat at 2.25% as in Exhibit 2, but volatility is 2%. Under this scenario, in which the only change from Exhibit 2 is the level of volatility, the Ho-Lee binomial lattice changes in two meaningful ways. First, the dispersion of possible future rate paths increases dramatically. Second, the normality assumption combined with the level of the term structure at 2.25% and 2% volatility results in negative interest rates. The possibility of negative rates results directly from the model's assumption of the normal distribution and is a highly undesirable characteristic. The next model we present also assumes the rate is distributed normally, but incorporates mean reversion of rates to control the potentially large growth that is possible in the Ho-Lee model.

The Hull-White Model: Incorporating Mean-Reversion

The model of Hull and White is similar to the Ho and Lee model, but begins with a different SDE (interest rate dynamic) selected to control the potentially large growth in the Ho-Lee model.[3] The Hull and White model aims to incorporate mean-reversion, which is an important and well-documented characteristic of interest rates. To incorporate mean-reversion, the drift term of the Hull-White model departs from that of the Ho-Lee model. The Hull-White SDE is as follows:

$$dr = (\theta - \phi r)dt + \sigma dz$$

Notice that the stochastic term is identical to the Ho-Lee. Thus, the possible future rates according to this model are also distributed normally and negative rates are again possible. The drift term is where the difference arises. Specifically, in the Hull-White model, the drift term does not grow unbridled as does the drift term in the Ho-Lee model. Instead, the drift term assumes there is a long-run mean and when the short rate deviates from that mean, "gravity" pulls it back. To illustrate, assume there is a long run mean interest rate, and we'll call it "μ." With a little mathematical trickery, we set $\mu=\theta/\phi$, and then the drift term becomes $\phi(\mu\text{-}r)$. Ignoring the potentially intimidating symbols, we will highlight that there are two important properties of the mean process worth mentioning here: there is a long-run mean interest rate, μ, to which the interest rate tends to revert, and the speed, ϕ, at which "gravity" pulls the short rate toward that mean. When the speed of mean reversion, ϕ, is positive, mean reversion controls the growth of the short rate, pulling it up (down) when below (above) the mean μ. Should negative mean reversion result, the short rate may grow exponentially. The conjunction of the parameters ϕ and μ determine the drift component.

[3] J. Hull and A. White, "Pricing Interest Rate Derivative Securities," Review of Financial Studies (1990), 3, pp. 573-592, and , "One Factor Interest Rate Models and the Valuation of Interest Rate Derivative Securities," Journal of Financial and Quantitative Analysis (1993), pp. 235-254.

Jumping over the details of compiling numerical solutions, the evolution of the short rate in the Hull-White model takes the following form[4]:

$$r_{k+1} = r_k + (\theta_k - \phi_k r_k)\tau + \sigma_k \varepsilon_k \sqrt{\tau}$$

Again, the normal distribution is evident and comes from the fact that the term ε_k is distributed normally, a similar assumption to the Ho-Lee model. Thus, the stochastics are distributed normally as the dynamic component consists of the volatility size multiplied by the normally distributed ε_k. The above equation states that the short rate in each sequential period equals yesterday's short rate, plus the drift term, plus the stochastic term. Under positive mean reversion, the drift term will be positive (negative) when the short rate r_k is below (above) the mean, pulling the short rate back toward the mean. Under negative mean-reversion, the rate will continue to be driven away from the mean. On average, the stochastic term is zero, since the mean of $\varepsilon_k$is zero. However, at each time step, the stochastic component accounts for the uncertainty regarding the forward evolution of the short rate, a critical component to pricing and risk management of bonds with embedded option features. Importantly, the Hull-White model improves upon the Ho-Lee model as it specifies more realistic drift term and, by controlling the growth rate, makes less likely the possibility that forward rates breach the zero barrier into negative territory.

The Kalotay-Williams-Fabozzi Model: Log-normally Distributed Rates

Similar to the Ho-Lee model, the Kalotay-Williams-Fabozzi interest rate model assumes the short rate follows a SDE with constant drift and volatility absent mean-reversion.[5] The incremental contribution, however, is that the model assumes that the logarithm of the short rate follows this process, whereas the Ho-Lee model assumes the rate itself follows this process. Thus, the SDE is:

$$d\ln(r) = \phi dt + \sigma dz$$

Although the natural log of the short rate may turn negative in this model, the rate itself does not turn negative. Thus, the model addresses a major shortcoming of models making the normal distribution assumption: that of negative interest rates. To illustrate, suppose that $\mu = ln(r)$. When $\mu < 0$, we can see that r is still non-negative by raising each side to the base e. This results in $r = e^{\mu}$, demonstrating that although r may become small and close to zero, it remains positive. Thus, the significant innovation from the Kalotay-Williams-Fabozzi model is that it addresses the negative rates problem associated with the Ho-Lee and Hull-White models.

Again skipping the details involved with obtaining a discrete form of the model, we present the following equation, which describes the evolution of the short rate through time according to the Kalotay-Williams-Fabozzi model[6]:

$$r_{k+1} = r_k e^{\theta_k \tau + \sigma_k \varepsilon_k \sqrt{\tau}}$$

The equation above indicates that the short rate grows unboundedly through time when $\theta > 0$. Conversely, if $\theta < 0$, then the short rate decays toward zero. Although the short rate may grow without bound, negative rates are not possible. Thus, relative to the normally distributed models, this model has the advantage of producing non-negative rates, but like the Ho-Lee model, suffers from potentially unbounded growth through time. Thus, in order to address the potential for unbounded growth through time, the next model we present combines the

[4] See Buetow, Gerald and James Sochacki, "Term Structure Models Using Binomial Trees," Research Foundation of AIMR (2001).
[5] A. Kalotay, G. Williams, and F.J. Fabozzi, "A Model for the Valuation of Bonds and Embedded Options," Financial Analysts Journal (May-June 1993), pp. 35-46.
[6] See Buetow, Gerald and James Sochacki, "Term Structure Models Using Binomial Trees," Research Foundation of AIMR (2001).

log-normal assumption which was the innovation used to address the negative forward rates, adding mean-reversion will address the potentially uncontrolled growth.

The Black-Karasinski Model: Log-normally Distributed Rates with Mean-Reversion

The Black-Karasinski model combines mean-reversion with the assumption that the short rate is distributed log-normally.[7] Specifically, the model assumes the log of the short rate, *ln(r)*, follows the following process:

$$d\ln(r) = \big(\theta - \phi\ln(r)\big)dt + \sigma dz$$

Where $r = ln(r)$, we note this process is the Hull-White process. Thus, the model combines the mean-reversion property of the Hull-White model, but avoids producing negative interest rates. Similar to the Hull-White model, the parameter ϕ controls the growth of the short rate, controlling the speed at which the natural log of the interest rate reverts back toward its mean.

In a similar fashion as we have done for the other models, we obtain a discrete version of the model describing the rate evolution from one period to the next[8]:

$$r_{k+1} = r_k e^{(\theta_k - \phi_k ln r_k)\tau + \sigma_k \varepsilon_k \sqrt{\tau}}$$

Again, we see that the drift in the short rate from period k to the next period is determined by the speed at which the rate reverts back to the mean, and the distance of the rate from that target rate. Additionally, although this model and the Hull-White model incorporate mean-reversion, the Black-Karasinski model, unlike the Hull-White model, assumes lognormality of the short rate, thus avoiding the undesirable property of producing negative rates.

Despite this model's sophistication and ability to overcome the two main drawbacks of the Ho-Lee model, there is still room for improvement. Specifically, although many models in finance assume that volatility is constant, in reality volatility is not constant. This fact is evident when comparing the volatility level implied from the prices of options on the same asset but having different times to expiration. Thus, we realize that the assumption that interest rate volatility is constant through time is an abstraction from reality. The next innovation we discuss incorporates the term structure of interest rate volatility.

The Black-Derman-Toy Model: Log-normally Distributed Rates with Endogenous Mean Reversion

The model of Black-Derman-Toy combines the properties of mean reversion and the lognormal distribution of the short rate.[9] Mean reversion, however, is determined based on the model's vector of inputs. As we will see shortly, the slope of the volatility curve determines the properties of mean reversion. The model well illustrates the tradeoff between complexity of the more parsimonious models and analytical tractability of the less restrictive. The Black-Derman-Toy model assumes the short rate follows the process:

$$d\ln(r) = \left(\theta(t) + \frac{\sigma'(t)}{\sigma(t)}\ln(r)\right)dt + \sigma(t)dz$$

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[7] F. Black and P. Karasinski, "Bond and Option Pricing when Short Rates are Lognormal," Financial Analyst Journal (July-August 1991), pp. 52-59.
[8] See Buetow, Gerald and James Sochacki, "Term Structure Models Using Binomial Trees," Research Foundation of AIMR (2001).
[9] F. Black, E. Derman, and W. Toy, "A One Factor Model of Interest Rates and Its Application to the Treasury Bond Options," Financial Analyst Journal (January-February 1990), pp. 33-39.

Where *σ(t)* is the volatility term structure. The model is similar to the Black-Karasinski model, and in fact, the only difference arises in the mean reversion term. The slope of the volatility curve, *σ'(t)*, plays an important role, influencing mean reversion in the model. When $\sigma'(t) < 0$, i.e. the volatility cure is downward sloping, the short rate mean reverts. Conversely, when $\sigma'(t) > 0$, i.e. the volatility cure is positively sloped, the short rate grows unboundedly. When $\sigma'(t) = 0$, the model reduces to the Kalotay, Williams and Fabozzi model.

The evolution of the rate across times k to $k+1$ is:

$$r_{k+1} = r_k e^{\left(\theta_k - \frac{\sigma'(t)}{\sigma(t)} ln r_k\right)\tau + \sigma_k \varepsilon_k \sqrt{\tau}}$$

The solution has a similar appearance to the Black-Karasinski model where interest rates are distributed log-normally, avoiding negative interest rates, and the short rate reverts to its mean through time. Of course the difference arises since the slope of the volatility curve determines mean reversion in this model.

Since the model of Black, Derman and Toy is the most sophisticated model presented here, we next illustrate the important properties of the model using binomial lattices. Our motivation is to highlight the differences between the BDT model, again the most sophisticated one-factor model we present here, and the Ho-Lee model. It is important to keep in mind that all of these models are no-arbitrage models, which means that valuation of option-free bonds conducted with the output of any of these models will result in the same answer—an answer that is consistent with the market prices of bonds. However, as we will highlight in a future commentary, the next installment in this series highlighting our approach to managing the fixed income exposures, the valuation results for bonds with embedded options depend critically upon the model selected. Exhibit 4 presents the BDT binomial lattice for a term structure flat at 2.25% and flat volatility curve at 10%.

Exhibit 4: BDT Binomial Lattice: Flat Term Structure, 10% Vol.

				2.76%
			2.58%	
		2.41%		2.40%
	2.25%		2.24%	
		2.09%		2.08%
			1.94%	
				1.81%
Time in Years	**0**	**0.5**	**1**	**1.5**

SOURCE: Innealta Capital

From Exhibit 4, the influence of the log-normal distribution is evident immediately. The rates are all positive and the distribution of future rates around the level of the term structure are asymmetric and are less dispersed compared to the Ho-Lee tree. Additionally, the same symmetry of the dispersion as was evident in the Ho-Lee model is not evident due to the lognormality.

Recall that mean-reversion of the short rate depends on the slope of the volatility term structure. Exhibit 5 presents the BDT lattice where the term structure is still flat at 2.25%, but the volatility curve is positively sloped, starting at 10% and increasing by 100 vol points per period.

Exhibit 5: BDT Binomial Lattice—Flat Term Structure, Negatively Sloped Vol Curve

				2.99%
			2.65%	
		2.41%		2.45%
	2.25%		2.23%	
		2.09%		2.01%
			1.89%	
				1.65%
Time in Years	**0**	**0.5**	**1**	**1.5**

SOURCE: Innealta Capital

For comparison purposes, the same term structure flat at 2.25%, but where the interest rate volatility term structure decreases by 1% each period, is presented below in Exhibit 6. The differences between the two are significant. The dispersion of the rates is greater for the positively sloped volatility curve, which reflects the fact that the positive slope of the vol curve results in unbounded growth in the short rate. In Exhibit 5 above, the downward sloped volatility curve leads to mean-reversion, controlling the growth of the short rate and acting as a gravitational force pulling the rates back toward the mean.

Exhibit 6: BDT Binomial Lattice—Flat Term Structure, Positively Sloped Vol Curve

				3.10%
			2.86%	
		2.57%		2.47%
	2.25%		2.22%	
		1.93%		1.97%
			1.72%	
				1.57%
Time in Years	**0**	**0.5**	**1**	**1.5**

SOURCE: Innealta Capital

SUMMARY AND CLOSING REMARKS

Our purpose this month has been to further illustrate some of the mechanics behind fixed income portfolio management, focusing on the task of modeling the term structure of interest rates. In the process, we have highlighted several of the most popular no-arbitrage models, emphasizing their distinctive properties and the ways in which those assumptions manifest themselves in the model output. Our presentation of the models, beginning with the most basic model, that of Ho and Lee, and ending with a more complex model, that of Black, Derman and Toy, exposes the tradeoff between analytical tractability and solutions that reflect the salient properties of interest rates. Each model presents an incremental improvement over the previous model, but also presents additional complexities when computing numerical solutions. Given the many term structure models available, it is imperative, in our view, that the manager recognizes the distinguishing characteristics of all available models before making investment decisions relying on any valuation or risk metrics.

In highlighting this class of interest rate models, we seek to demonstrate to our readers and clients some of the tools we use to understand how the macroeconomic actions, from monetary policy to fiscal uncertainty, propagate their way through the system and ultimately manifest in the term structure—and, ultimately, the bond valuations—through the interpretations of these actions by other market participants. Importantly, the review of these methods only skims the great depth of work we perform when analyzing our opportunity sets, both in fixed income and in equities.

An intended byproduct of this series of missives, too, is the understanding by readers that Innealta is well versed in navigating these waters. Noted in many commentaries over the past few years, we remain confident that we won't soon (measured in years) see any strong upward pressures on U.S. interest rates. Even so, we remain cognizant and vigilant of the sources of those pressures, ready to alter the income-related investments in the portfolio as the indications present. Meantime, we can utilize a breadth of methodologies, a small component of which we have presented this month, to test scenarios in advance to be sure we can pull the trigger quick as we need to when evolving circumstances so demand.

ENVIRONMENT AND POSITIONING

Though we made no changes to the Sector Rotation Portfolio, which retains a 10% exposure to the Energy sector, we added four country markets to the Country Rotation Portfolio in September, bringing total equity exposure in that strategy to 25%. Country markets now represented in the portfolio are Columbia, Hong Kong, Russia, Singapore and South Africa. Aside from prorating the holdings with the CRP, we otherwise left alone the fixed income allocations within both strategies.

Both portfolios in our view necessarily remain quite defensive, despite the added equity. Finding few additional markets—among either the U.S. sectors or other countries—currently sparking interest, we're inclined to view the long list of existing and potentially growing risks the nation and the world will confront as we move into the fall as reasons to stay mostly on the sidelines for the time being. Or longer...

Almost needless to say, we have a very important election coming up, the various potential results of which are very likely to include a wide range of consequences for our country's fiscal and macroeconomic future. Meantime, European policymakers and regulators continue to headfake resolutions to the Eurozone's debt and growth woes, failing to actually follow through with any true progress to stated goals. For their parts, Brazil, China and India seem more likely to find slower growth on the horizon. And the war of words between Israel and Iran seems evermore threatening to become worse than that, while the rest of the Middle East simmers in a stew of popular contempt, devoid of leadership.

Against that backdrop, and confirmed by our mostly bearish review of the dashboard of results from our quantitative framework, we can't help but think the summer's run in equities was built so much more on hope than fundamentals. Participants in such breathless runs, in our view, are likely to fall flat on their faces. We'd rather hold back, keeping a steadier, more sustainable pace, and win in the end by not having lost the discipline and control demanded by such perilous courses.

IMPORTANT INFORMATION

The information provided comes from independent sources believed reliable, but accuracy is not guaranteed and has not been independently verified. The security information, portfolio management and tactical decision process are opinions of Innealta Capital (Innealta), a division of Al Frank Asset Management, Inc. and the performance results of such recommendations are subject to risks and uncertainties. For more information about Al Frank Asset Management please visit afamcapital.com. Past performance is not a guarantee of future results.

Any investment is subject to risk. Exchange traded funds (ETFs) are subject to risks similar to those of stocks, such as market risk, and investors that have their funds invested in accordance with the portfolios may experience losses. Additionally, fixed income (bond) ETFs are subject to interest rate risk which is the risk that debt securities in a portfolio will decline in value because of increases in market interest rates. The value of an investment and the return on invested capital will fluctuate over time and, when sold or redeemed, may be worth less than its original cost. This material is not intended as and should not be used to provide investment advice and is not an offer to sell a security or a solicitation or an offer, or a recommendation, to buy a security. Investors should consult with an investment advisor to determine the appropriate investment vehicle. Investment decisions should be made based on the investor's specific financial needs and objectives, goals, time horizon and risk tolerance. All opinions and views constitute our judgments as of the date of writing and are subject to change at any time without notice.

Sector ETFs, such as Real Estate Investment Trusts ("REITs") are subject to industry concentration risk, which is the chance that stocks comprising the sector ETF will decline due to adverse developments in the respective industry.

The use of leverage (borrowed capital) by an exchange-traded fund increases the risk to the fund. The more a fund invests in leveraged instruments, the more the leverage will magnify gains or losses on those investments.

Country/Regional risk is the chance that world events such as political upheaval or natural disaster will adversely affect the value of securities issued by companies in foreign countries or regions. Country/Regional risk is especially high in emerging markets.

Emerging markets risk is that chance that stocks of companies located in emerging markets will be substantially more volatile, and substantially less liquid, than the stocks of companies located in more developed foreign markets.

Securities rated below investment grade, commonly referred to as "junk bonds", may involve greater risks than securities in higher rating categories. Junk bonds are regarded as speculative in nature, involve greater risk of default by the issuing entity, and may be subject to greater market fluctuations than higher rated fixed income securities.

Diversification does not protect against loss in declining markets.

Registration of an investment adviser does not imply any certain level of skill or training.

Al Frank Asset Management, Inc. is an Investment Adviser, registered with the Securities & Exchange Commission and notice filed in the State of California and various other states. For more information, please visit afamcapital.*com*.

Innealta is an asset manager specializing in the active management of portfolios of Exchange Traded Funds. Innealta's competitive advantage is its quantitative investment strategy driven by a proprietary econometric model created by Dr. Gerald Buetow, Innealta's Chief Investment Officer. The firm's products include Tactical ETF Portfolios, a U.S. Sector Rotation Portfolio and a Country Rotation Portfolio. Innealta aims to beat appropriate benchmark performance by tactically managing portfolios utilizing a proprietary econometric model. By harnessing the benefits of ETFs, Innealta is able to provide investors with exposure to multiple asset classes and investment styles in highly liquid, low cost portfolios.

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